

Demonstration of quantum error correction for enhanced sensitivity of photonic measurements

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The sensitivity of classical and quantum sensing is impaired in a noisy environment. Thus, one of the main challenges facing sensing protocols is to reduce the noise while preserving the signal. State of the art quantum sensing protocols that rely on dynamical decoupling achieve this goal under the restriction of long noise correlation times. We implement a proof-of-principle experiment of a protocol to recover sensitivity by using an error correction for photonic systems that does not have this restriction. The protocol uses a protected entangled qubit to correct a single error. Our results show a recovery of about 87% of the sensitivity, independent of the noise rate.

PACS numbers: 03.67.Bg, 06.20.Dk, 03.67.Pp

I. INTRODUCTION

Error correction is an essential ingredient in classical computation and communication and is currently used in many state of the art technologies. Shor's discovery of an algorithm [1] to break the RSA cryptosystem showed that a quantum computer is a promising system for tackling hard computational problems. However, it was not clear if even conceptually a quantum computer could be constructed. The reason for the large doubt, is that even a small error in every computational step would accumulate quite swiftly to a large error. It implies that the future quantum computer could solve only small and probably trivial problems. Remarkably, the theory of quantum fault tolerance [2–5] showed that this intuition is wrong. Actually, what is needed for the calculation of a quantum computer to give the right result with a small probability of failure is that every gate operation will fail with a small probability below a certain threshold. Put simply, a gate error below the threshold is effectively as good as no error at all. To date, many quantum error correction schemes have been suggested [1, 3, 4, 6–9] and implemented [10–14]. The predicted ability of a fault tolerant quantum computer to reach any desired precision raises a question whether this remarkable precision could be used for precise measurements.

Quantum metrology [15]; i.e., enhanced metrology using quantum mechanics, is a well developed field with applications to photonic interferometry [16, 17], magnetometry [18–20] and atomic spectroscopy [21]. However, the quantum advantage rapidly degrades as noise takes its toll [22]. Therefore, to maintain this advantage in a noisy environment, error correction needs to be applied. Several theory papers have been devoted to this issue that deal with a variety of noise types [23–29] and there has been one implementation with NV centers in diamond [30] using a carbon nuclear spin as the redundant qubit

and a related implementation [31] using the NV nuclear spin as the redundant qubit.

In this work, we implement a combination of error correction and sensing for the first time using a photonic platform. The protocol we implement uses only linear optical elements. The error correction relies on the use of a protected entangled photon qubit and a gate which is realized by post-selection. Not only the phase information, which we are interested in, is preserved, but the entire quantum state as well. Due to the use of post-selection, each error correction cycle is only 50% viable; however, this setup provides a proof of principle experiment that could have been achieved with deterministic gates which are currently being developed [32–35].

II. THEORETICAL BACKGROUND

Let us consider a quantum state evolving under a Hamiltonian which depends on an unknown parameter, which we want to measure. The state changes continuously as a function of the parameter as it propagates through a noisy path vulnerable to a single type of error, either a bit-flip or a phase-flip [23, 24]. If the data are integrated without an error correction, sensitivity drops as a result of cancellation between measurements with and without the error. However, if an error correction protocol is applied much faster than the time at which the deleterious effects of the noise become significant, the sensitivity level can be maintained. This scenario can be simulated by repeating a series of parameter probing and noise operations many times. Instead of measuring once over a time T , as in the usual sensing scheme, a measurement of duration $\frac{T}{N}$ is repeated N times, where between each measurement the error is corrected.

We present and implement the protocol by the optical realization of qubits using the polarization degree of freedom of photons, and denote the vertical (horizontal) linear polarization as $|V\rangle$ ($|H\rangle$). The birefringence phase, denoted as θ , is the unknown parameter. The scheme is composed of a regular interferometric setup for measur-

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ing a birefringent phase [36]. However, in the presented model, the measurement of the birefringent phase is also associated with a noisy environment that flips the qubit. Although we chose to correct bit-flips, phase-flip errors could have been corrected instead with a minor addition of polarization rotators.

III. EXPERIMENTAL SETUP AND STATE EVOLUTION

The full error correction scheme is presented in Fig. 1. First, the Bell state

$$|\phi^+\rangle = \frac{1}{\sqrt{2}} (|H\rangle_1 |H\rangle_2 + |V\rangle_1 |V\rangle_2), \quad (1)$$

is generated, where the first qubit, denoted by $|\cdot\rangle_1$, is protected and the second, denoted by $|\cdot\rangle_2$ and used for the phase measurement, is not. After the phase measurement the state is

$$|\phi^\theta\rangle = \frac{1}{\sqrt{2}} (|H\rangle_1 |H\rangle_2 + e^{i\theta} |V\rangle_1 |V\rangle_2). \quad (2)$$

If the noise is applied, the state becomes a mixture of the original state and

$$|\psi^\theta\rangle = \hat{\sigma}_I \otimes \hat{\sigma}_x |\phi^\theta\rangle = \frac{1}{\sqrt{2}} (|H\rangle_1 |V\rangle_2 + e^{i\theta} |V\rangle_1 |H\rangle_2). \quad (3)$$

To correct the error, a phase of $\frac{\pi}{2}$ is added to the two qubits and a Hadamard transform is operated solely on the second. Then, the two photons are directed to the two inputs of a polarizing beam splitter (PBS). Only the events in which the two output paths of the PBS are both occupied by a photon are post-selected, thus guaranteeing that both post-selected photons are either horizontally or vertically polarized. After the error correction operation, both the perturbed and unperturbed states return to

$$|\phi^\theta\rangle, |\psi^\theta\rangle \xrightarrow{\text{correction}} |\phi^\theta\rangle = \frac{1}{\sqrt{2}} (|H\rangle_{1'} |H\rangle_{2'} + e^{i\theta} |V\rangle_{1'} |V\rangle_{2'}). \quad (4)$$

Here the entanglement is between the two spatial paths, $1'$, the un-delayed spatial path and, $2'$, the delayed spatial path.

Error correction is realized by employing a non-unitary operation using a PBS and post-selection. This is analogous to applying an error correction without measurement [37] and initialization. The difference here is that this is realized without extending the size of the code, but by applying a decoding operation.

In order to repeat the measurement, one photon is measured while its twin (denoted as $|\cdot\rangle_2^0$) is delayed for time δ until another pair of entangled photons, $\frac{1}{\sqrt{2}} (|H\rangle_1^\delta |H\rangle_2^\delta + |V\rangle_1^\delta |V\rangle_2^\delta)$, is generated, where the upper index denotes the time of creation. After the second pair accumulates the phase and is corrected, the

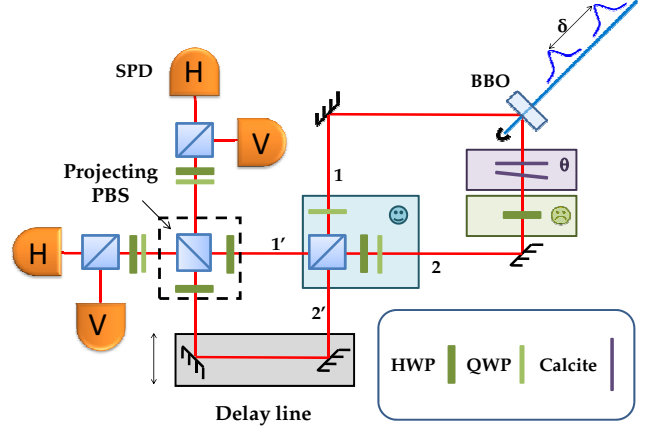


FIG. 1. (color online). The experimental setup. 780 nm entangled photons are generated by a non-collinear type-II parametric down-conversion from 2 mm thick β -BaB₂O₄ (BBO) crystal using a 390 nm doubled Ti:Sapphire pulsed laser [39]. One photon measures a birefringence phase (θ in the figure), generated by two 2 mm tilted Calcite crystals with perpendicular optical axes [36]. The bit-flip is generated by a 45° half waveplate (HWP) for the horizontal polarization (sad smiley in the figure). The error correction is composed of two quarter waveplates (QWPs) placed in the two paths and a 22.5° HWP in the noisy path. A PBS completes the error correction protocol. The multiple phase accumulation part is implemented by a 31.5 m long delay line, projecting photons from different pulses [38]. Two 22.5° HWPs are placed before the projecting PBS to avoid the collapse of the state by the measurement of the first photon. The block of the projecting PBS and the HWPs was not used, as the results in this work were taken without the multiple phase accumulation part. Eventually, basis transformation is carried out with HWPs and QWPs and the photons are detected by single photon detectors. The data are accumulated by FPGA electronics.

delayed photon from the first pair interferes on a PBS with the un-delayed photon (denoted as $|\cdot\rangle_{1'}^\delta$) from the second pair, swapping the entanglement and resulting in the state $\frac{1}{\sqrt{2}} (|H\rangle_{1'}^0 |H\rangle_{2'}^\delta + e^{i2\theta} |V\rangle_{1'}^0 |V\rangle_{2'}^\delta)$. After $N - 1$ similar iterations the state is

$$|\phi^{N\theta}\rangle = \frac{1}{\sqrt{2}} (|H\rangle_{1'}^0 |H\rangle_{2'}^{(N-1)\delta} + e^{iN\theta} |V\rangle_{1'}^0 |V\rangle_{2'}^{(N-1)\delta}), \quad (5)$$

which oscillates N times faster than the state of single iteration. Thus, in a scenario in which the gate would have been realized deterministically, the sensitivity would have been increased by a factor of \sqrt{N} compared to the shot-noise limit, the limit of classical measurements. More experimental details and a discussion on the entanglement swapping between delayed photons can be found in Ref. [38].

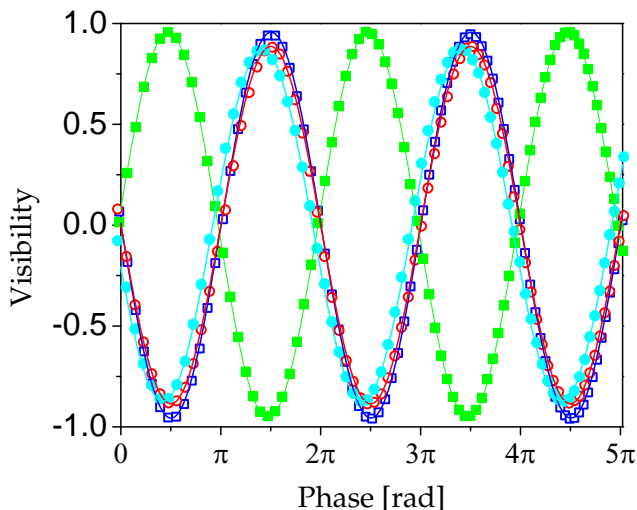


FIG. 2. (color online). The visibility V_{RL} as a function of the phase. Solid and empty symbols denote perturbed and unperturbed states, respectively. Circles and boxes denote corrected and uncorrected states, respectively. Solid lines represent fits Sine functions. The corrected states have the same phase as the original state, while the perturbed uncorrected state has an opposite phase. Errors were calculated assuming Poisson error distributions. They are smaller than the symbol sizes, and thus they are not displayed.

IV. RESULTS

We divide the demonstration into two parts: correction of bit-flip errors and the successive accumulation of consecutive measurement results. The second part has already been demonstrated by our group in a previous work [38]; here we demonstrate the error correction part.

A. Single phase accumulation

The phase information can be retrieved by interfering the H/V amplitudes. One way to achieve this is to detect the photons in the circular polarization basis. A right (left) handed circularly polarized photon is defined as $|R\rangle(|L\rangle) = \frac{1}{\sqrt{2}}(|H\rangle_{(-)}^{\pm} i|V\rangle)$. Figure 2 shows the results of the phase dependent interference, where the visibility of the state in the circular basis V_{RL} is presented as a function of the phase. The V_{RL} visibility is defined as $P_{RR} - P_{RL} - P_{LR} + P_{LL}$, where for example P_{RL} denotes the probability for coincidence of a right and a left circularly polarized photon. A visibility as high as 94% was observed in the case of the original (unperturbed uncorrected) state, which indicates a well prepared initial state, where the corrected states have slightly lower visibilities due to optical imperfections. The phases of the corrected states are similar to the original phase, but the phase of the perturbed uncorrected state is not.

Our demonstration provides an important example

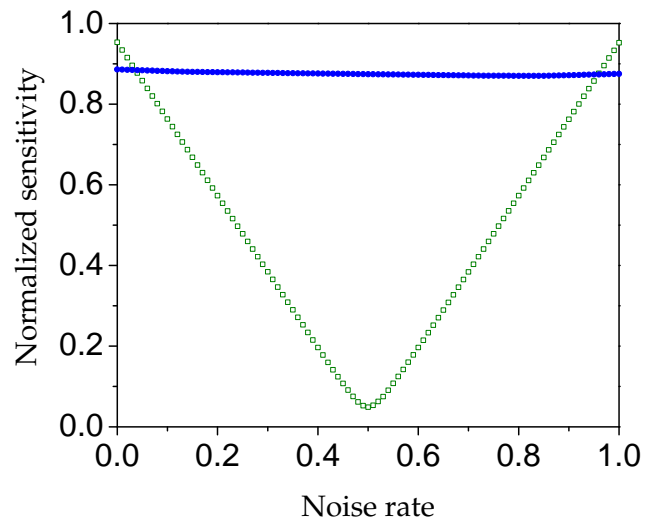


FIG. 3. (color online). The sensitivity as a function of the noise rate, with an error correction (solid blue circles) and without (empty green boxes). Errors were calculated assuming Poisson error distributions. They are smaller than the symbol sizes, and thus they are not displayed.

where using an entangled (quantum) state has an advantage over a separable (classical) state [25]. The entanglement allows for the correction of the state and preservation of the measurement of the phase without an error. In Fig. 3, we used the experimental results presented in Fig. 2 to estimate the sensitivity as a function of the noise rate. The count number is normalized, making the maximal sensitivity unity. The sensitivity after the error correction is greater than 0.87 and nearly independent of the noise rate. The sensitivity without error correction is linearly impaired and almost vanish as the noise rate achieves 0.5. In the presented scenario, in principle, the correction could restore the fidelity completely as the sensing of the birefringent phase and the bit-flip noise are spatially disconnected and are induced in the right order, i.e., the signal first and then the bit-flip. Noteworthy is that in other scenarios in which both the noise and the signal overlap in time or in space, or occur in different order, a large rate of error correction is required to reach high sensitivity. Nevertheless, when the signal measurement and noise stages are repeated multiple times, the significance of their order vanishes with the increasing number of stages.

B. Towards multiple phase accumulations

As mentioned, the projection on the PBS is used to accumulate the phase repeatedly. It works only if the projection is between two $|\phi^\theta\rangle$'s, which means that the entire state must be corrected. Thus, we also performed a complete quantum state tomography to the perturbed/unperturbed corrected/uncorrected states for all

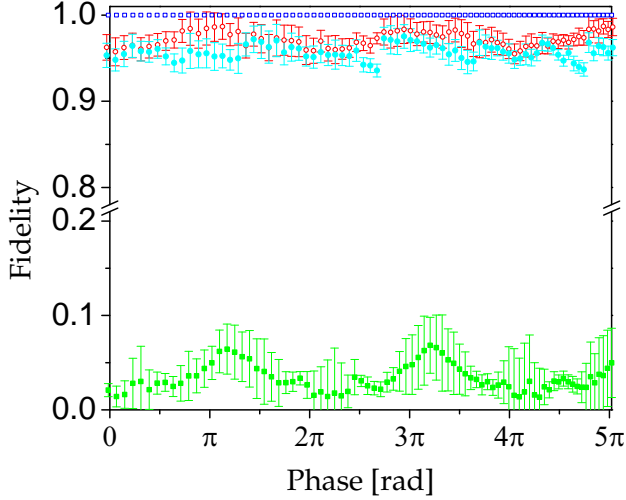


FIG. 4. (color online). The fidelity with the original state as a function of the phase. The notations and colors are the same as in Fig. 2. Errors were calculated assuming Poisson error distributions.

phases [40]. We have calculated the fidelities between all states and the original state [41], which is presented in Fig. 4. The fidelities are close to one for all phases which indicates that we have recovered the original state. In contrast, the fidelity with the perturbed uncorrected state is close to zero, demonstrating the orthogonality between these states.

V. CONCLUSIONS

In conclusion, we have realized a proposal to use a quantum error correction protocol to recover sensitivity in the presence of a single type error such as bit-flipping noise. A linear optics implementation of the measurement and the error correction is presented and demonstrated using a PBS and post-selection on the polarization degree of freedom of photons. The perturbed state was projected back onto the original state. The results show a significant sensitivity difference between the corrected and uncorrected states, where more than 87% of the visibility is preserved, independently of the noise rate.

VI. APPENDIX

A. General noise

First we would like to present the error correction protocol for a more general noise. The noise rotation that occurs with probability p is

$$\begin{aligned} |H\rangle &\xrightarrow{\text{error}} |\psi\rangle = \alpha|H\rangle + \beta|V\rangle, \\ |V\rangle &\xrightarrow{\text{error}} |\psi^\perp\rangle = e^{i\epsilon}(\beta^*|H\rangle - \alpha^*|V\rangle). \end{aligned} \quad (\text{a.6})$$

where, without loss of generality, we assume that $\beta > 0$ and real. The entangled perturbed state is $|\phi^\theta\rangle_{\text{noised}} = \frac{1}{\sqrt{2}}(|H\rangle_1|\psi\rangle_2 + e^{i\theta}|V\rangle_1|\psi^\perp\rangle_2)$. The experiment that was realized is a specific example for $\beta = 1$ and the phase $\epsilon = 0$.

We now prove that the transformation,

$$\hat{U} = \begin{pmatrix} \cos \eta & e^{i\xi} \sin \eta \\ -e^{-i\xi} \sin \eta & \cos \eta \end{pmatrix} \quad (\text{a.7})$$

on the noisy qubit and the applied post-selection correct the error, where $\cos \eta = \frac{\beta}{\sqrt{\beta^2 + |e^{i\chi} - \alpha|^2}}$, $\chi = \frac{\epsilon - \pi}{2}$ and ξ is the phase of the non-zero complex number $e^{i\chi} - \alpha$. This transformation can be generated in the lab by a phase shifter, a half-wave plate at angle η and another phase shifter. It can be noticed that

$$\begin{aligned} \langle H|\hat{U}|H\rangle &= \langle V|\hat{U}|V\rangle = \cos \eta, \\ \langle H|\hat{U}|\psi\rangle &= \alpha \cos \eta + \beta e^{i\xi} \sin \eta, \\ \langle V|\hat{U}|\psi^\perp\rangle &= -e^{i\epsilon}(\alpha^* \cos \eta + \beta e^{-i\xi} \sin \eta). \end{aligned} \quad (\text{a.8})$$

The second and third terms are simplified by taking a common factor of $\cos \eta$. Then, we replace $\beta \tan \eta$ by $|e^{i\chi} - \alpha|$ but ξ is the phase of this number. Thus, α (α^*) is cancelled out and we get:

$$\begin{aligned} \langle H|\hat{U}|H\rangle &= \langle V|\hat{U}|V\rangle = \cos \eta, \\ \langle H|\hat{U}|\rho\rangle &= e^{i\chi} \cos \eta, \\ \langle V|\hat{U}|\rho^\perp\rangle &= e^{i\chi} \cos \eta. \end{aligned} \quad (\text{a.9})$$

Where in the third expression the definition for χ is used.

Next, only the events where the two photons have the same polarization are post-selected via a PBS. Therefore, $|\psi\rangle \rightarrow e^{i\chi} \cos \eta |H\rangle$ and $|\psi^\perp\rangle \rightarrow e^{i\chi} \cos \eta |V\rangle$ and we get

$$\begin{aligned} |\phi^\theta\rangle &\xrightarrow{\hat{U} + \text{postselection}} \cos \eta |\phi^\theta\rangle, \\ |\phi^\theta\rangle_{\text{perturbed}} &\xrightarrow{\hat{U} + \text{postselection}} e^{i\chi} \cos \eta |\phi^\theta\rangle. \end{aligned} \quad (\text{a.10})$$

The state is corrected up to a global phase χ with a probability of $\cos^2 \eta$.

B. Full scheme derivation

Using deterministic gates the setup is composed of a gate that maps the noise to one qubit and the information to the other, followed by an initialization of the qubit that contains the noise [30]. A more detailed explanation is needed where a PBS and post-selection is used, where one can think that the post-selection for the error correction and entanglement swapping are mixed. The reason it does work is due to post-selection both in space and time.

The derivation is divided into three steps: First, we show that for a four photon event without noise the final state is

$$\frac{1}{\sqrt{2}}(|H\rangle_1^0|H\rangle_{2'}^\delta + e^{i2\theta}|V\rangle_1^0|V\rangle_{2'}^\delta), \quad (\text{a.11})$$

where as in the text the upper index represents the time of creation and the lower index represents the spatial path. Second, we will generalize it to a four photon event with or without noise. Third, we will generalize this to a $2N$ photon event.

We assume that two pairs of photons were generated, and we neglect high order terms. We only look at sequences where one photon is detected at time zero, two photons are detected at time $t = \delta$ (in different spatial paths) and one photon is detected at time $t = 2\delta$. δ is the travel time of a photon in the delay line. After the measurement and correction of the first pair (i.e. after the PBS and before the post-selection) the state is:

$$\frac{1}{\sqrt{2}}(|H\rangle_{1'}|H\rangle_{2'} + e^{i\theta}|V\rangle_{1'}|V\rangle_{2'} + |H\rangle_{2'}|V\rangle_{2'} - e^{i\theta}|V\rangle_{1'}|H\rangle_{1'}) \quad (\text{a.12})$$

The last two terms contribute to two photons at time $t = 0$ or $t = \delta$. If the two photons are measured in different detectors, the post-selection fails and data are disregarded. If the two photons are measured with the same detector as one photon, the sequence will be a three photon event or less, and thus will be discarded. Therefore, the temporal post-selection only leaves the two first terms, where one photon will be detected at time $t = 0$ and one at time $t = \delta$. Thus, in order to get the right sequence, another photon must be detected at time $t = \delta$ and its twin at time $t = 2\delta$. This can only happen if the second pair is generated at a delay of δ with respect to the generation of the first pair and exactly one photon enters the delay line. If these conditions are fulfilled, the four photon state is

$$\frac{1}{2}(|H\rangle_{1'}^0|H\rangle_{2'}^\delta + e^{i\theta}|V\rangle_{1'}^0|V\rangle_{2'}^\delta) \times (|H\rangle_{1'}^\delta|H\rangle_{2'}^{2\delta} + e^{i\theta}|V\rangle_{1'}^\delta|V\rangle_{2'}^{2\delta}) \quad (\text{a.13})$$

The next step is entanglement swapping on a PBS. We first define $|\phi^\theta\rangle = \cos\frac{\theta}{2}|\phi^+\rangle - i\sin\frac{\theta}{2}|\phi^-\rangle$ (up to a global phase). Then, two half-wave plates rotate the basis from H/V to P/M , the 45° diagonal linear polarization basis. After returning to the H/V basis, Eq. a.13 becomes

$$\left(\cos\frac{\theta}{2}|\phi^+\rangle_{1',2'}^{0,\delta} - i\sin\frac{\theta}{2}|\psi^+\rangle_{1',2'}^{0,\delta}\right) \times \left(\cos\frac{\theta}{2}|\phi^+\rangle_{1',2'}^{\delta,2\delta} - i\sin\frac{\theta}{2}|\psi^+\rangle_{1',2'}^{\delta,2\delta}\right), \quad (\text{a.14})$$

where the double index refers to the first and second photons in the Bell state. We consider only the events where the delayed photon of the first pair (denoted by $|\cdot\rangle_{2'}^\delta$) and the non-delayed photon of the second pair (denoted by $|\cdot\rangle_{1'}^\delta$) are in different paths after the PBS, which gives:

$$\begin{aligned} |\phi^+\rangle_{1',2'}^{0,\delta}|\phi^+\rangle_{1',2'}^{\delta,2\delta} &\rightarrow |\phi^+\rangle_{1',2'}^{0,2\delta} \\ |\phi^+\rangle_{1',2'}^{0,\delta}|\psi^+\rangle_{1',2'}^{\delta,2\delta} &\rightarrow |\psi^+\rangle_{1',2'}^{0,2\delta} \\ |\psi^+\rangle_{1',2'}^{0,\delta}|\phi^+\rangle_{1',2'}^{\delta,2\delta} &\rightarrow |\psi^+\rangle_{1',2'}^{0,2\delta} \\ |\psi^+\rangle_{1',2'}^{0,\delta}|\psi^+\rangle_{1',2'}^{\delta,2\delta} &\rightarrow |\phi^+\rangle_{1',2'}^{0,2\delta}. \end{aligned} \quad (\text{a.15})$$

Using basic trigonometric identities we get the state, $\cos\theta|\phi^+\rangle_{1',2'}^{0,2\delta} - i\sin\theta|\psi^+\rangle_{1',2'}^{0,2\delta}$. Another rotation by a half-wave plate results in the state of Eq. a.11, as was required.

The generalization to a perturbed state is straightforward. The error correction post-selection is done by post-selecting the time of photon detections. This post-selection is independent from the noise. After this post-selection, the state is corrected back to $|\phi^\theta\rangle$. Thus, the entanglement swapping is between two corrected states, and the result is the same.

Finally, a similar derivation applies to any number of photon pairs. An event of one photon in time zero, two photons in time $t = \delta, 2\delta, \dots, (N-2)\delta$ and one photon in time $(N-1)\delta$ can only be detected if all the error correction post-selections succeeded. Then, the entanglement swapping is always between two states of $|\phi^\theta\rangle$ (with different θ 's), which results in the state

$$\frac{1}{\sqrt{2}}(|H\rangle_{1'}^0|H\rangle_{2'}^{(N-1)\delta} + e^{iN\theta}|V\rangle_{1'}^0|V\rangle_{2'}^{(N-1)\delta}) \quad (\text{a.16})$$

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